

SM3 9.5 Solving Trig Equations

Solve each equation over the interval $[0, 2\pi)$.

1) $2 \cos \theta + 4 = 5$

$$2 \cos \theta = 1$$

$$\cos \theta = \frac{1}{2}$$

$$\theta = \left\{ \frac{\pi}{3}, \frac{5\pi}{3} \right\}$$

2) $2 \sin \theta - 1 = 0$

$$2 \sin \theta = 1$$

$$\sin \theta = \frac{1}{2}$$

$$\theta = \left\{ \frac{\pi}{6}, \frac{5\pi}{6} \right\}$$

3) $\tan^2 \theta - 3 = 0$

$$\tan^2 \theta = 3$$

$$\tan \theta = \pm\sqrt{3}$$

$$\theta = \left\{ \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3} \right\}$$

4) $5 \cos \theta - \sqrt{3} = 3 \cos \theta$

$$2 \cos \theta = \sqrt{3}$$

$$\cos \theta = \frac{\sqrt{3}}{2}$$

$$\theta = \left\{ \frac{\pi}{6}, \frac{11\pi}{6} \right\}$$

5) $4 \sec^2 \theta - 2 = 0$

$$4 \sec^2 \theta = 2$$

$$\sec^2 \theta = \frac{1}{2}$$

$$\cos^2 \theta = 2$$

$$\cos \theta = \pm\sqrt{2}$$

$$\theta = \emptyset$$

6) $\sin^2 \theta - 5 \cos \theta = 5$

$$(1 - \cos^2 \theta) - 5 \cos \theta = 5$$

$$\cos^2 \theta + 5 \cos \theta + 4 = 0$$

$$(\cos \theta + 1)(\cos \theta + 4) = 0$$

$$\cos \theta = -1, \quad \cos \theta = -4$$

$$\theta = \pi$$

7) $4 \sin^2 \theta - 2 = 0$

$$4 \sin^2 \theta = 2$$

$$\sin^2 \theta = \frac{1}{2}$$

$$\sin \theta = \pm \frac{\sqrt{2}}{2}$$

$$\theta = \left\{ \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4} \right\}$$

8) $3 \tan \theta - \sqrt{3} = 0$

$$3 \tan \theta = \sqrt{3}$$

$$\tan \theta = \frac{\sqrt{3}}{3}$$

$$\theta = \left\{ \frac{\pi}{6}, \frac{7\pi}{6} \right\}$$

$$9) \sec(\theta) + 2 = 0$$

$$\sec(\theta) = -2$$

$$\cos(\theta) = -\frac{1}{2}$$

$$\theta = \left\{ \frac{2\pi}{3}, \frac{4\pi}{3} \right\}$$

$$10) \sin^2 \theta - 4 \sin \theta = 5$$

$$\sin^2 \theta - 4 \sin \theta - 5 = 0$$

$$(\sin \theta - 5)(\sin \theta + 1) = 0$$

$$\sin \theta = 5, \quad \sin \theta = -1$$

$$\theta = \frac{3\pi}{2}$$

$$11) \cot \theta \sec \theta + \cot \theta = 0$$

$$\frac{\cos \theta}{\sin \theta} \frac{1}{\cos \theta} + \frac{\cos \theta}{\sin \theta} = 0$$

$$\frac{1}{\sin \theta} + \frac{\cos \theta}{\sin \theta} = 0$$

$$\frac{1 + \cos \theta}{\sin \theta} = 0$$

$$1 + \cos \theta = 0$$

$$\cos \theta = -1$$

$$\theta = \pi$$

Fails to check: $\theta = \emptyset$

$$12) 5 \cos 2\theta + 1 = 3 \cos 2\theta$$

$$2 \cos 2\theta = -1$$

$$2(2 \cos^2 \theta - 1) = -1$$

$$4 \cos^2 \theta - 2 = -1$$

$$\cos^2 \theta = \frac{1}{4}$$

$$\cos \theta = \pm \frac{1}{2}$$

$$\theta = \left\{ \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3} \right\}$$

Solve each equation over the interval $(-2\pi, 2\pi)$:

$$13) 16 \cos^2 \theta - 8 = 0$$

$$\cos^2 \theta = \frac{1}{2}$$

$$\cos \theta = \pm \frac{\sqrt{2}}{2}$$

$$\theta = \left\{ -\frac{7\pi}{4}, -\frac{5\pi}{4}, -\frac{3\pi}{4}, -\frac{\pi}{4}, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4} \right\}$$

$$14) 2 \cos^2 \theta - 3 \cos \theta + 1 = 0$$

$$(2 \cos \theta - 1)(\cos \theta - 1) = 0$$

$$2 \cos \theta - 1 = 0, \quad \cos \theta - 1 = 0$$

$$\cos \theta = \frac{1}{2}, \quad \cos \theta = 1$$

$$\theta = \left\{ -\frac{5\pi}{3}, -\frac{\pi}{3}, 0, \frac{\pi}{3}, \frac{5\pi}{3} \right\}$$